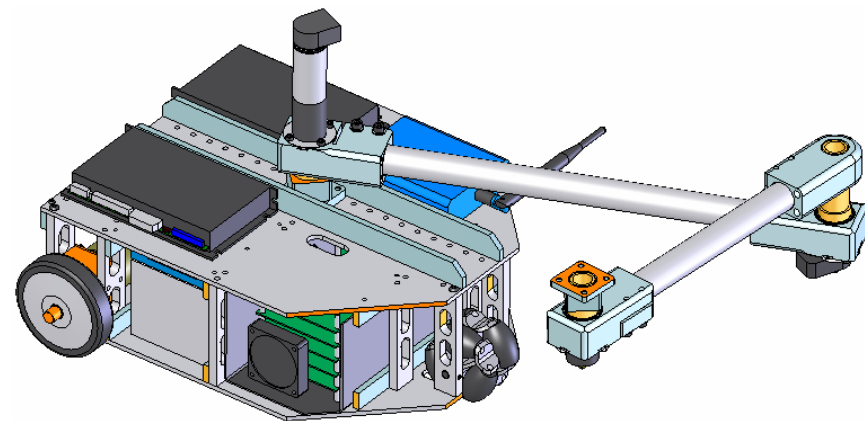
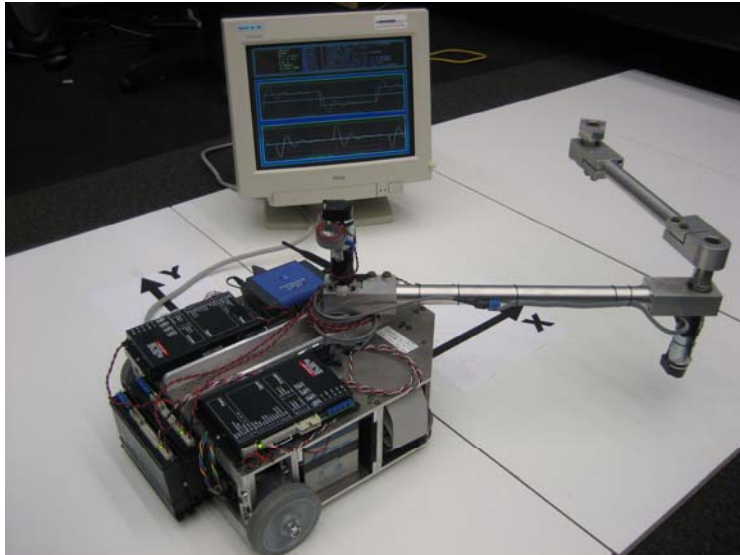


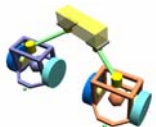
# Decoupled Dynamic Control of a Nonholonomic Wheeled Mobile Manipulators



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Glenn D. White

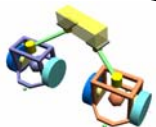


# Motivation

*Mobile manipulator systems are useful in extending the reach of humans in many manipulation & environment interaction tasks.*



**Applications: Payload manipulation, arc welding, highway maintenance, mining, explosive/hazardous materials disposal, etc.**



# Advantages

## **Combined DOF can reach much larger workspace.**

- **Manipulator** is typically more dexterous and can perform finer motion with greater performance.
- **Mobile base** provides the mobility and carries the load of the manipulator so that it can reach any point in the plane.

## **Redundancy in configuration.**

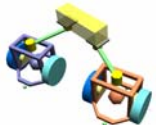
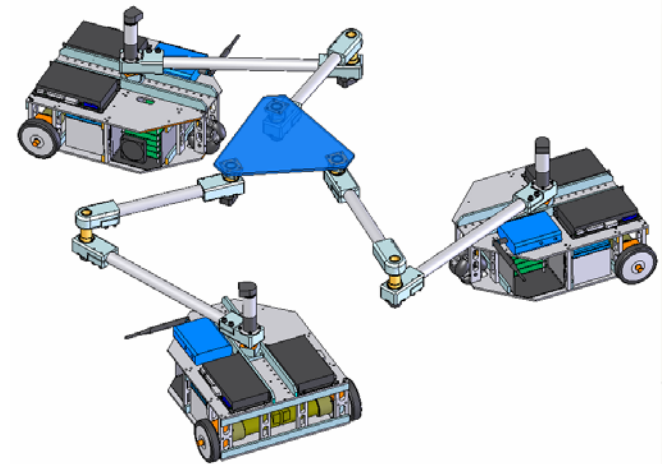
- Flexibility to perform more complex and different tasks.
- Ability to mechanically accommodate, detect, and compensate for disturbances.

## **Our ultimate goal:**

Cooperative payload transport by multiple wheeled mobile manipulators.

## **Each module =**

Differentially-driven wheeled mobile robot  
+ planar RR manipulator



# Research Issues

## Design:

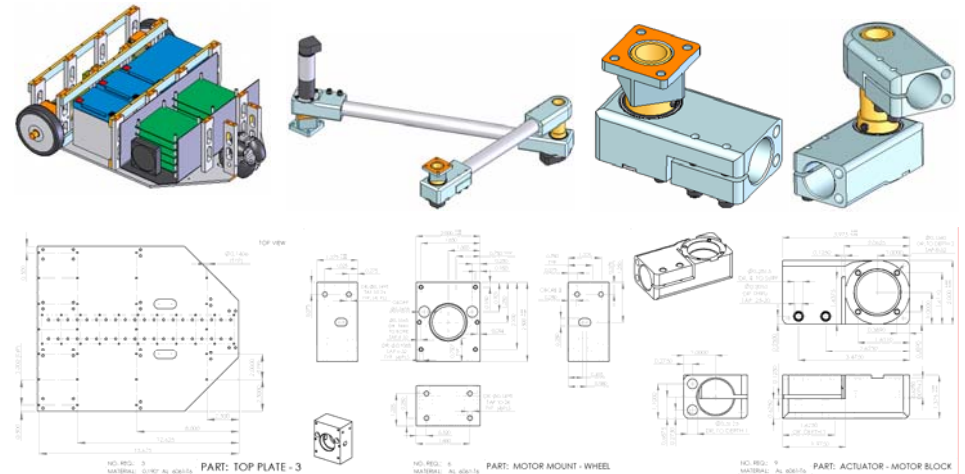
- Topology, dimensions, actuation
- Physical prototype realization

## Control:

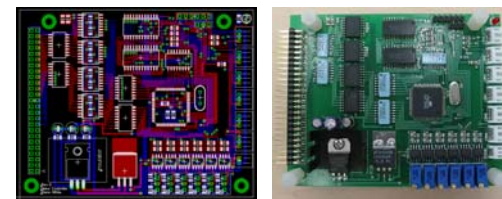
- Redundancy resolution schemes (kinematic/dynamics)
- Disturbance compensation/rejection

## Validation:

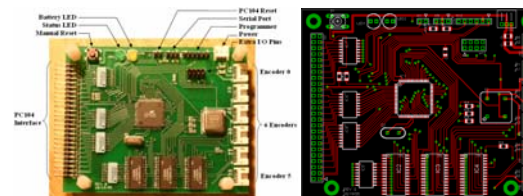
- Electronics subsystems design
- Hardware-in-the-loop (HIL)
- Virtual prototyping/co-simulations



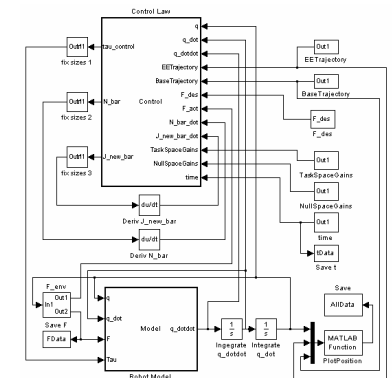
**Mechanical Prototype Realization**



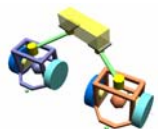
**Motor Controller**



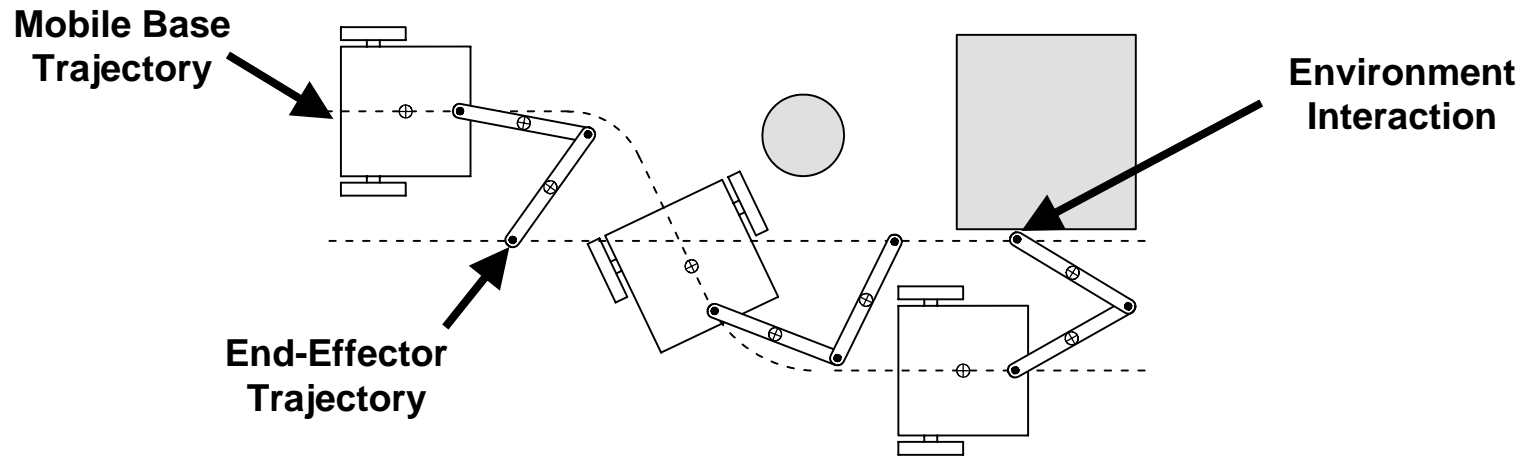
**Encoder Coprocessor**



**Redundancy Resolution Algorithms**

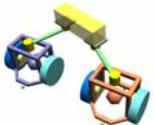


## Scenario



## Challenges

- *Nonholonomic constraints* due to wheel assemblies.
- The *dynamic interaction* between the manipulator and the mobile base is highly coupled (in both motions and forces)
- Combining mobility of the wheeled platform and mounted manipulator creates both *kinematic and actuator/dynamic redundancy*.

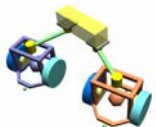
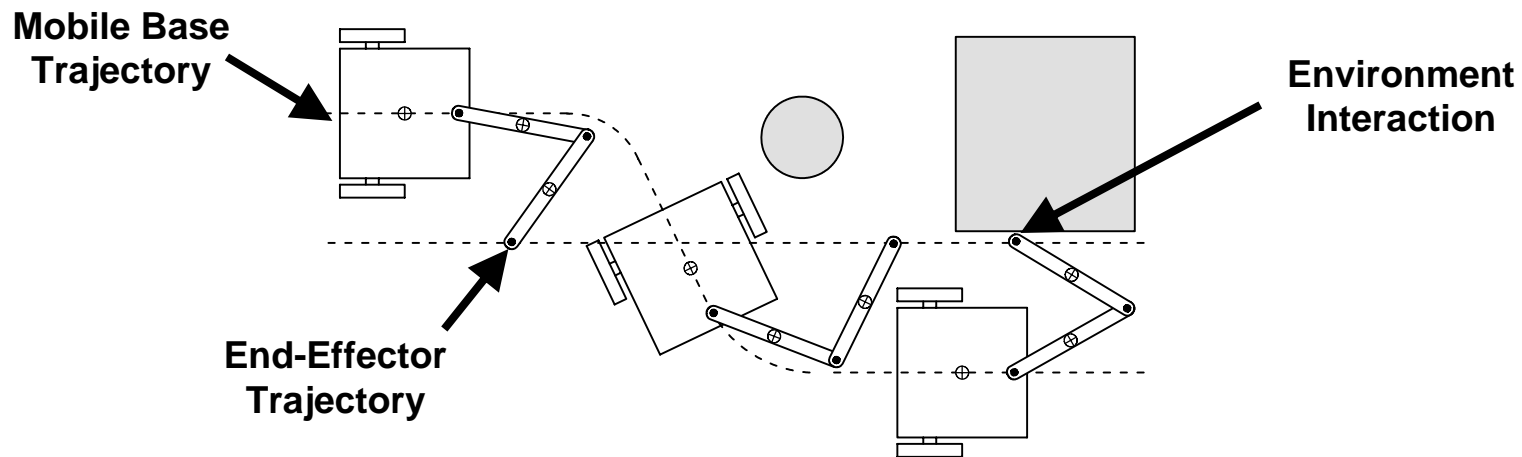


# Our Focus

Decoupling the system dynamics into **task-space** and **null-space** components...

...such that:

- the *primary* and *secondary objectives* can be achieved independently in a *dynamically consistent sense*
- ...while taking into account of *nonholonomic constraints*.



# Dynamic Modeling

**Constrained Dynamic Equation**  
with extended generalized coordinates

$$M(\underline{q})\ddot{\underline{q}} + V(\underline{q}, \dot{\underline{q}}) = E\tau_m + E_2\underline{F} - A^T \underline{\lambda}$$

$$\underline{q} = [x_c \ y_c \ \phi \ \theta_R \ \theta_L \ \theta_1 \ \theta_2]^T$$

**Nonholonomic Constraints**

$$A(\underline{q})\dot{\underline{q}} = 0$$

$$S^T \xrightarrow{\text{Projection}} S^T A^T = 0$$

**Feasible Dynamic Equation**  
with minimal generalized coordinates

$$S^T M S \dot{\underline{\nu}} + S^T M \dot{S} \underline{\nu} + S^T V = S^T E \tau_m + S^T E_2 \underline{F}$$

$$\underline{\nu} = [\dot{\theta}_R \ \dot{\theta}_L \ \dot{\theta}_1 \ \dot{\theta}_2]^T$$

**Standard Form (via Change of Variables):**

$$H \ddot{\underline{z}} + C \dot{\underline{z}} + g = \underline{\tau} + \underline{\tau}_E$$

$$\dot{\underline{z}} = \underline{\nu}, \ddot{\underline{z}} = \dot{\underline{\nu}}$$

**External (End-Effector) Forces**

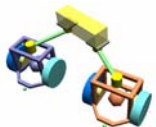
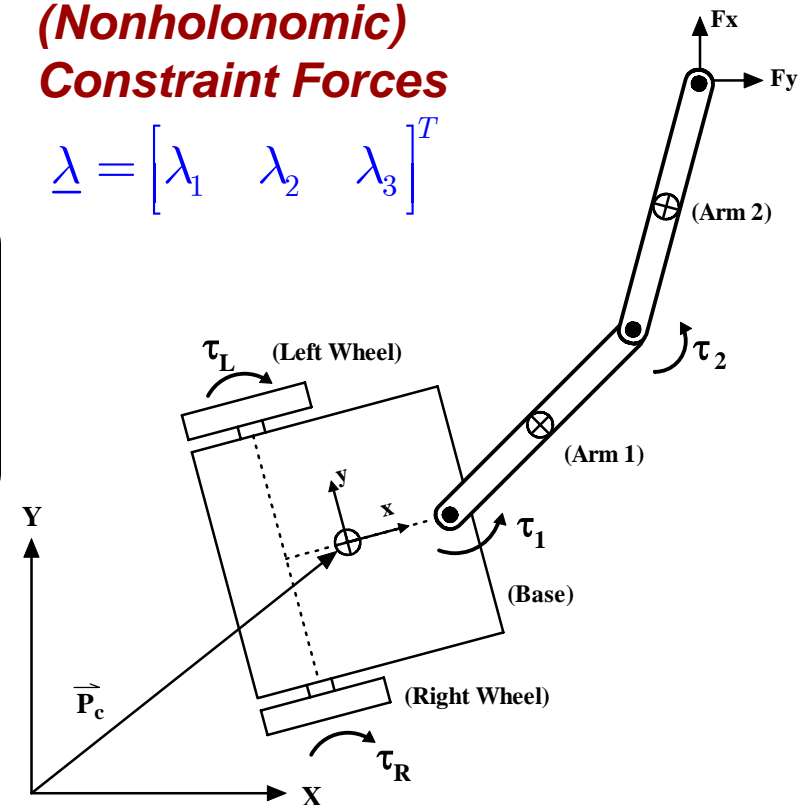
$$\underline{F} = [F_x \ F_y]^T$$

**Joint Actuations**

$$\tau_m = [\tau_R \ \tau_L \ \tau_1 \ \tau_2]^T$$

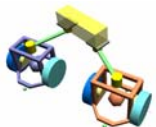
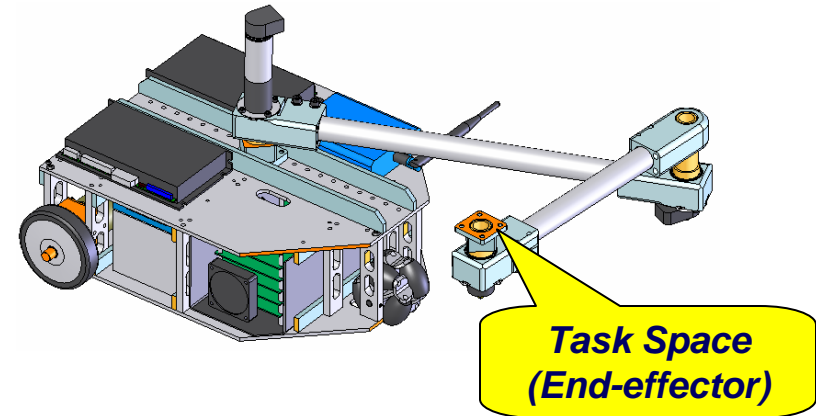
**(Nonholonomic) Constraint Forces**

$$\underline{\lambda} = [\lambda_1 \ \lambda_2 \ \lambda_3]^T$$



# Redundancy Resolution Approach

- Identifying two spaces:
  - **Task-space** (End-effector)
  - **(Internal) configuration-space** (Mobile base configuration)
- **Projection** of dynamic equations into decoupled task- and null-spaces ensuring dynamic consistency
- Creates inputs for each using **nonlinear feedback**
- Extends methods for redundant holonomic manipulators to the mobile manipulators
- Advantages:
  - Development and linear superposition of **independent task- and null-space controllers**.
  - **Guaranteeing task achievement** in the event of the task-space and configuration-space **trajectories conflict**.



# Dynamic Redundancy Resolution

Mapping between  
joint space ( $n$  dim)

to

task space ( $p$  dim)

$$\underline{\dot{x}}_p = J \underline{\dot{z}}_n$$

## Kinematic Redundancy Resolution Schemes

Velocity (rate) level  $\underline{\dot{z}} = J^\# \underline{\dot{x}} + N \underline{\dot{z}}_h$

Acceleration level  $\underline{\ddot{z}} = J^\# (\underline{\ddot{x}} - \underline{\dot{J}} \underline{\dot{z}}) + N \underline{\ddot{z}}_h$

Special Case: Take “Dynamically Consistent Pseudoinverse”

$$J^\# = \bar{J} = H^{-1} J^T (J H^{-1} J^T)^{-1}$$

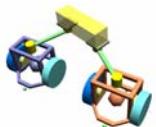
$$N^T = \bar{N}^T = I - J^T \bar{J}^T$$

Decoupled (Minimal) Joint Space Dynamics:

$$J^T \underline{F} + \bar{N}^T \underline{\tau} = \underbrace{J^T \bar{J}^T (H \bar{J} (\underline{\ddot{x}} - \underline{\dot{J}} \underline{\dot{z}}) + C \underline{\dot{z}} + \underline{g} - \underline{\tau}_E)}_{\text{Task Dynamic}} + \underbrace{\bar{N}^T (H \bar{N} (\underline{\ddot{z}}) + C \underline{\dot{z}} + \underline{g} - \underline{\tau}_E)}_{\text{Null (Internal) Dynamic}} + \underbrace{J^T \bar{J}^T H \bar{N} \underline{\ddot{z}} + \bar{N}^T H \bar{J} (\underline{\ddot{x}} - \underline{\dot{J}} \underline{\dot{z}})}_{\text{Coupling}}$$

Total Input Torque:

$$\underline{\tau} = \underbrace{J^T W (\underline{u} - \underline{\dot{J}} \underline{\dot{z}})}_{\text{Task Space Control}} + \underbrace{\bar{N}^T H (\underline{v} + \underline{\dot{J}} \underline{\dot{x}})}_{\text{Null (Internal) Space Control}} + C \underline{\dot{z}} + \underline{g} + J^T \underline{F}_E$$



# Task and Internal Space Control

The control laws for task space and null (internal) space can now be designed independently.

## Task-Space Control [Primary Control Space]

- External task space control that primarily achieves the tracking of end-effector motion/force with performance guarantees.

### Task Space Dynamics:

$$W(\underline{z})\ddot{\underline{x}} + \underline{\mu}(\underline{z}, \dot{\underline{z}}) + \underline{\gamma}(\underline{z}) = \underline{F} + \underline{F}_E$$

### Motion Objective:

$$\underline{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \rightarrow \begin{bmatrix} x_{end-effector} \\ y_{end-effector} \end{bmatrix}$$

### Task Impedance Control Law (Simultaneous Motion/Force Control):

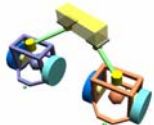
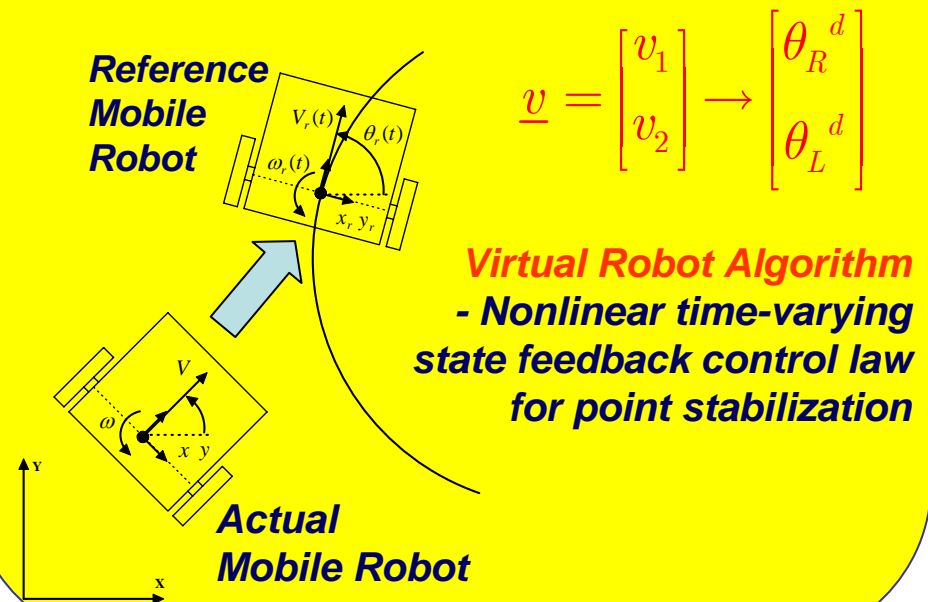
$$\underline{u} = \ddot{\underline{x}}_d + k_v \dot{\underline{e}} + k_p \underline{e} + k_f (\underline{F}_d - \underline{F}_E)$$

### Stable Error Dynamics:

$$\ddot{\underline{e}} + k_v \dot{\underline{e}} + k_p \underline{e} = k_f (\underline{F}_d - \underline{F}_E)$$

## Null-(Internal) Space Control [Secondary Control Space]

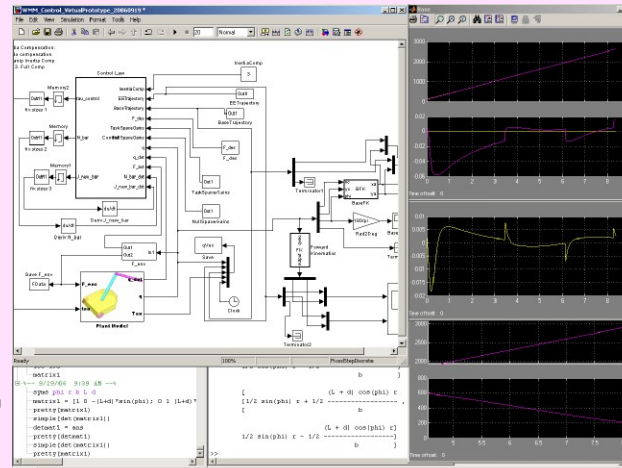
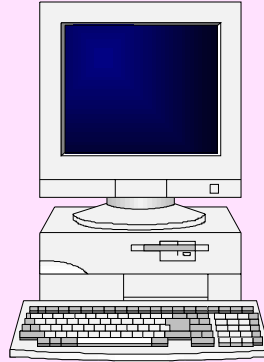
- Internal redundant control that secondarily attempts to achieve mobile base tracking of a reference trajectory.



# Integrated Software Framework

Distributed  
Virtual  
Prototyping

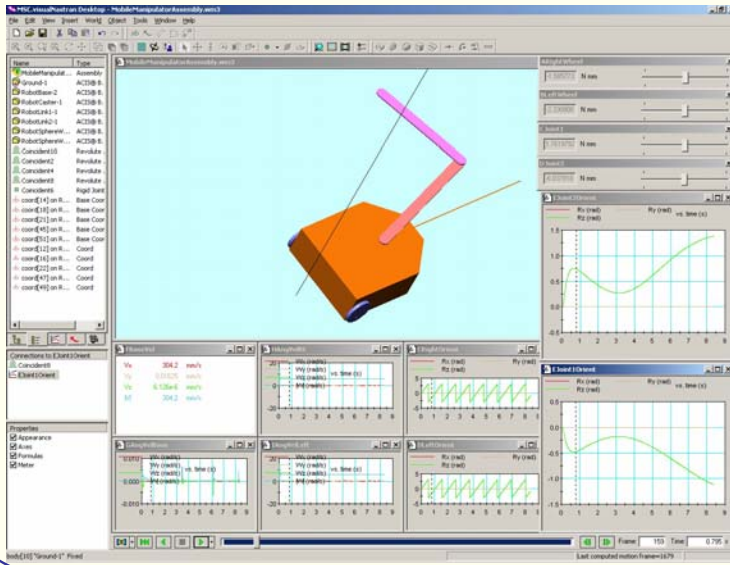
Ethernet/TCPIP



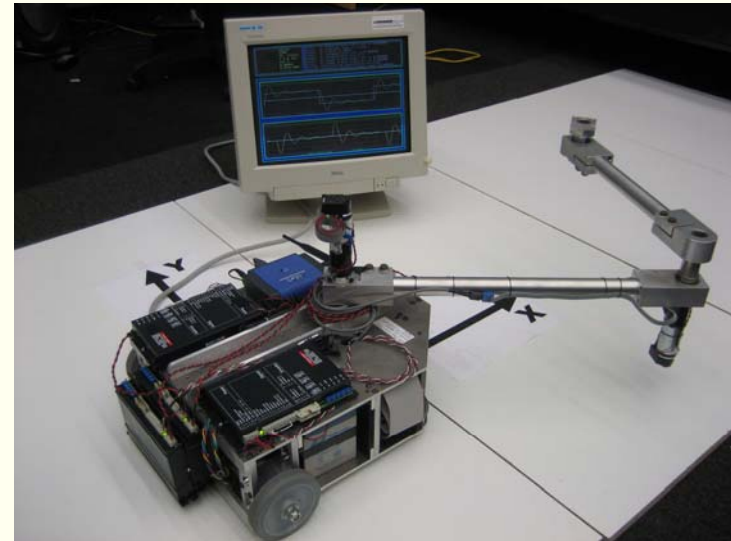
Remote  
Distributed  
Control

Ethernet/TCPIP

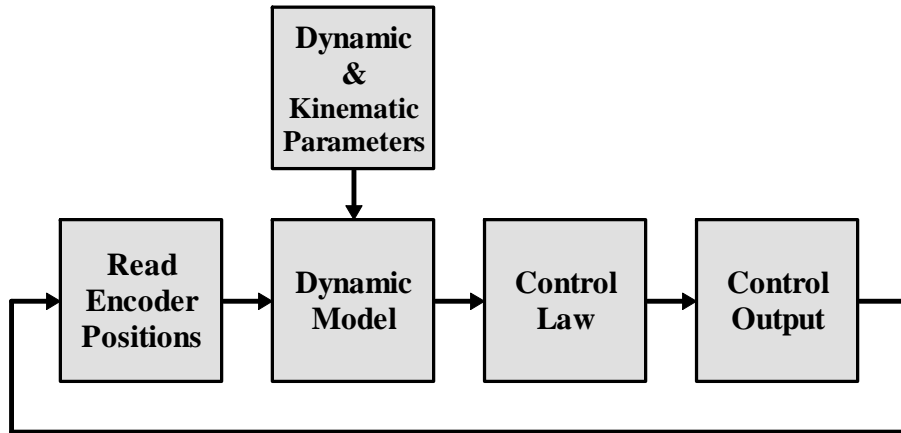
## Virtual Prototype Simulation-based refinement



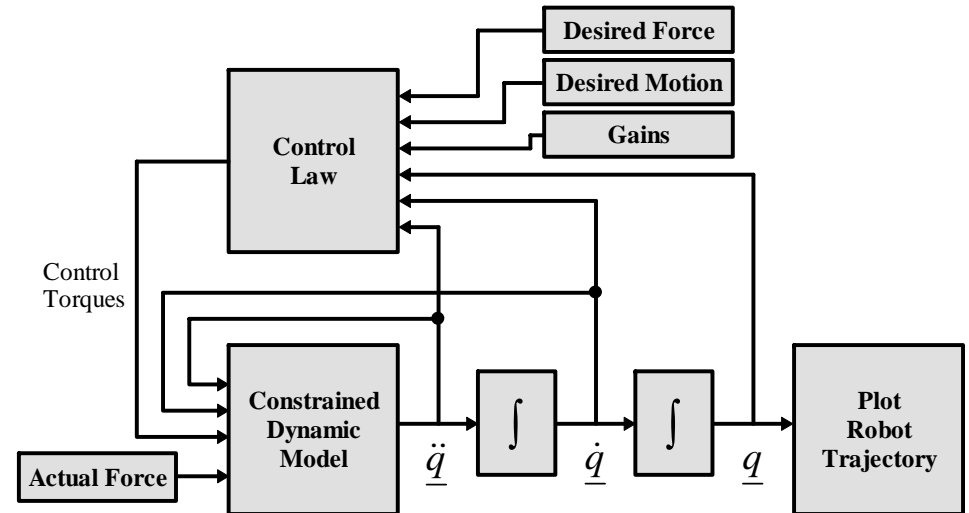
## Physical Prototype Hardware-in-the-loop testing



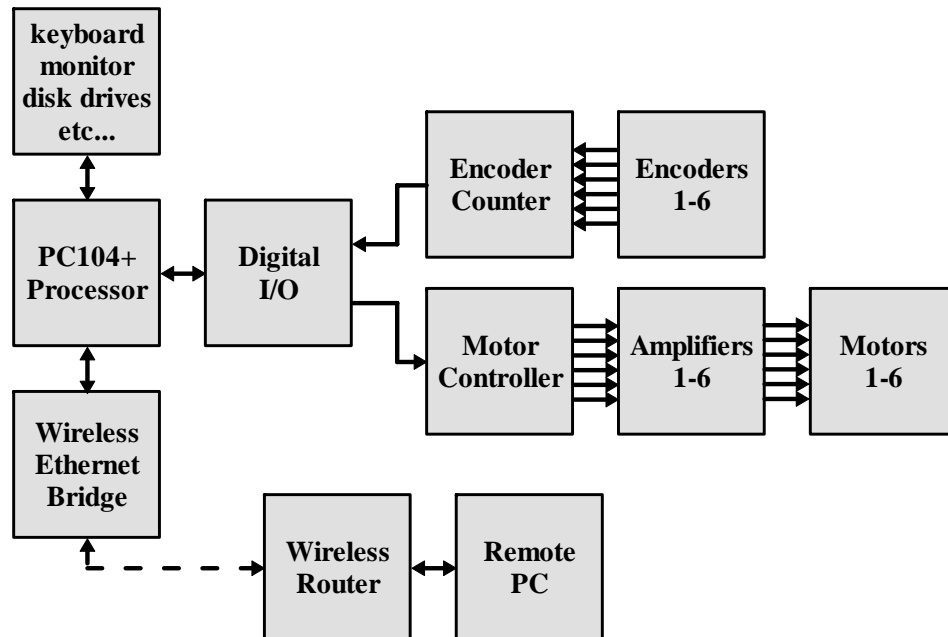
# Implementation Framework



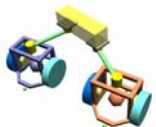
**High Level System Implementation**



**Software/Control Algorithm Implementation (MATLAB/Simulink)**

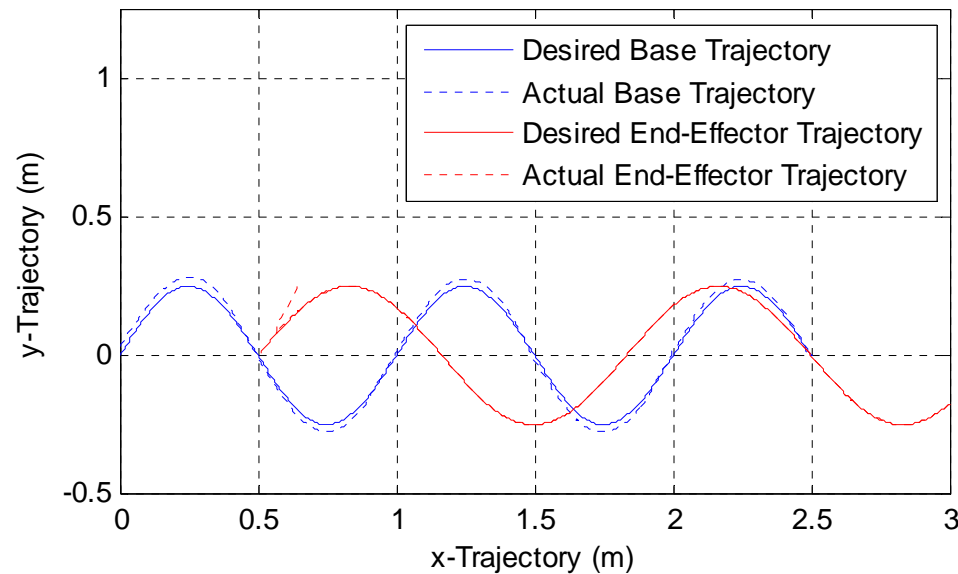


**PC/104-Based Electronics Hardware Implementation**



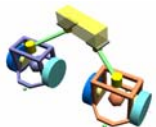
# Simulation I: Motion / Trajectory Tracking

**End-effector and mobile base  
follow separate sinusoids  
No interaction force control**



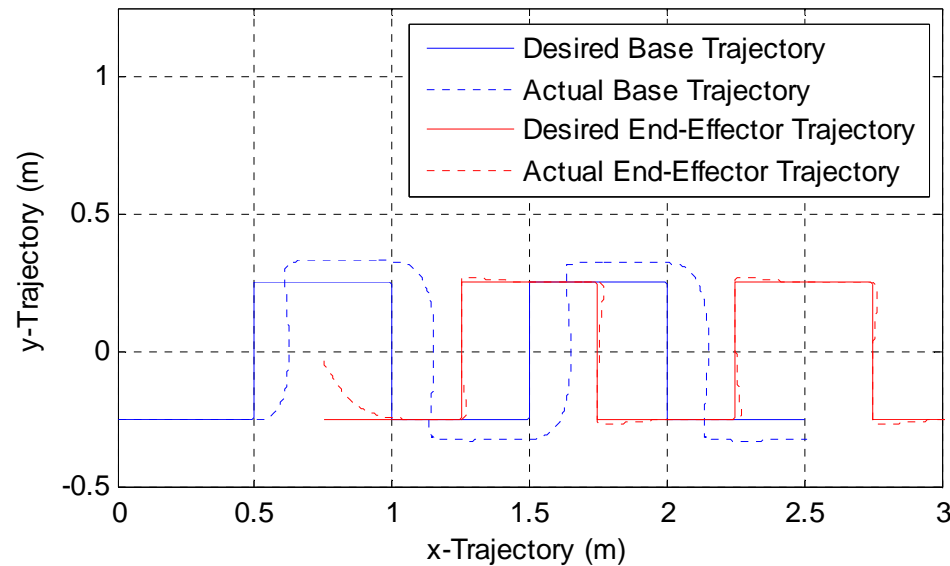
[Movie](#)

- Initial errors at both end-effector and mobile base are corrected.
- There are no conflicting control objectives. Both end-effector and mobile base track the desired trajectory closely.



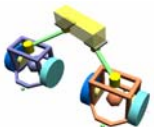
# Simulation II: Motion / Trajectory Tracking

**End-effector and mobile base  
follow separate square waves  
No interaction force control**



[Movie](#)

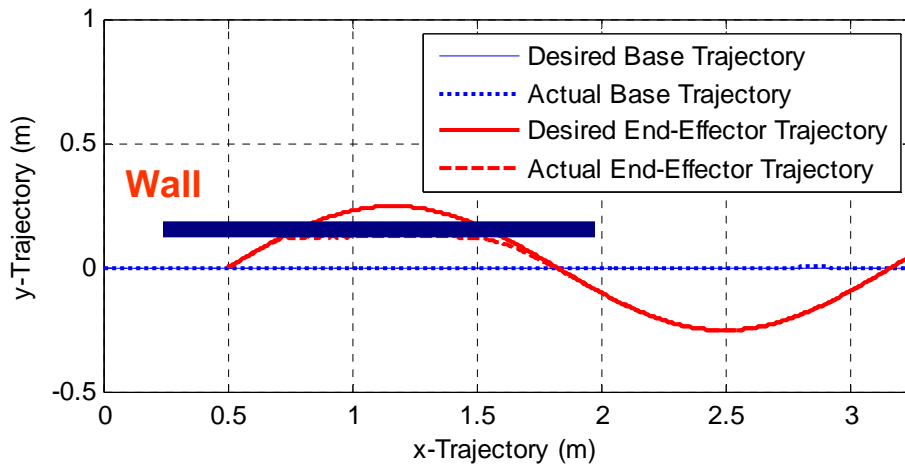
- Initial errors at the end-effector is corrected.
- The control objectives of the end-effector and mobile base are conflicting during operation. End-effector tracks the required trajectory closely, but mobile base attempts to track “approximately”.



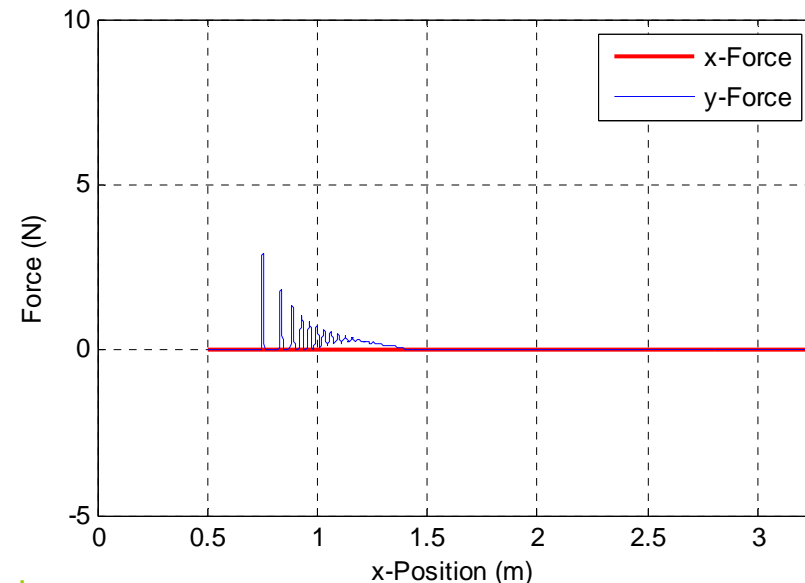
# Simulation III: Simultaneous Motion / Interaction Force Tracking

*End-effector follows sinusoids and applying desired (zero) force at a wall*  
*Mobile base follow a straight line trajectory*

Trajectory

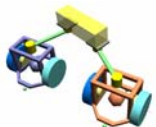


Force



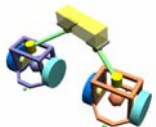
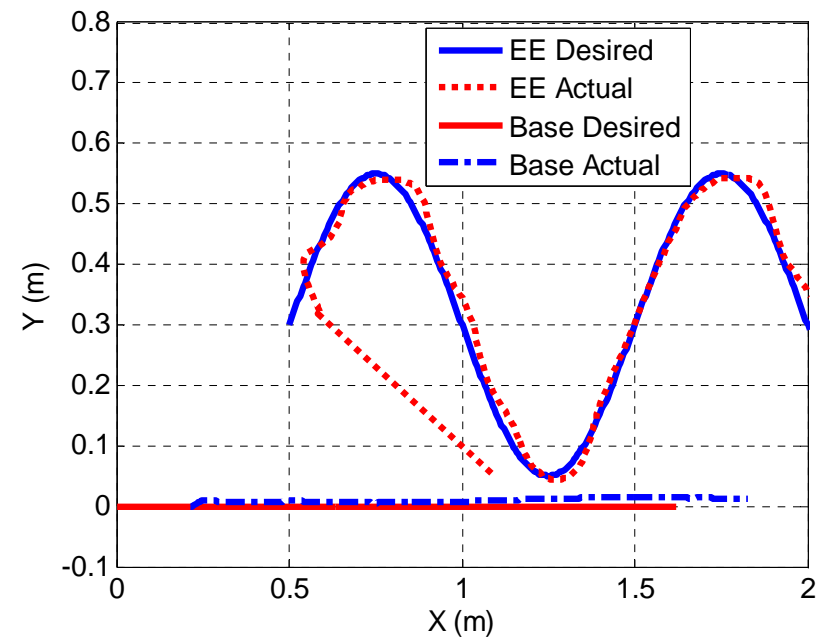
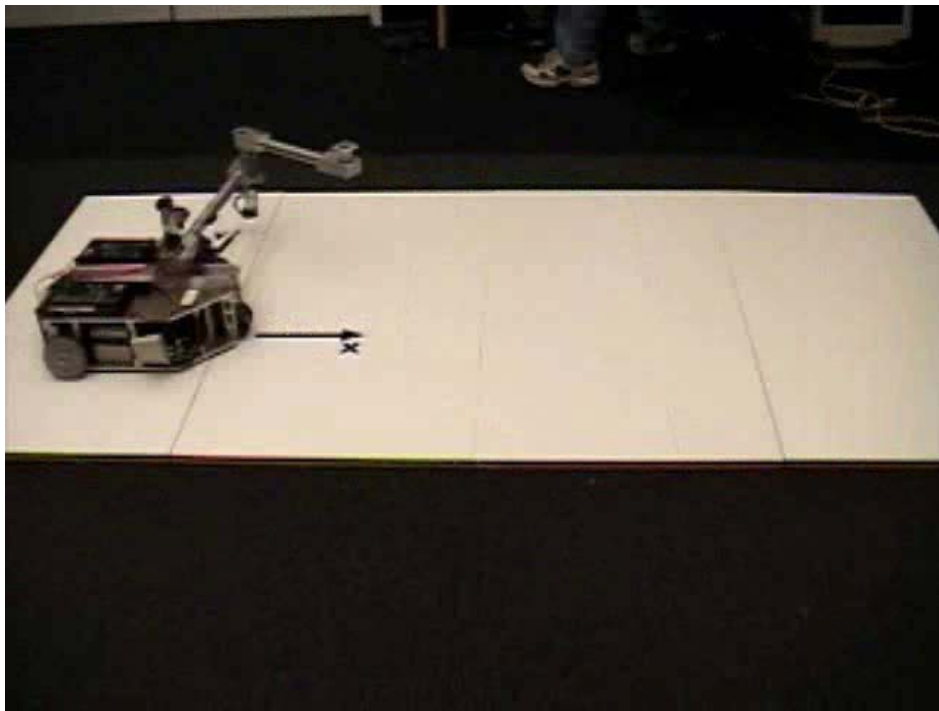
[Movie](#)

- When the end-effector hits the wall, the system attempt to apply the desired (constant/zero) force at the wall.



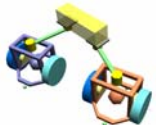
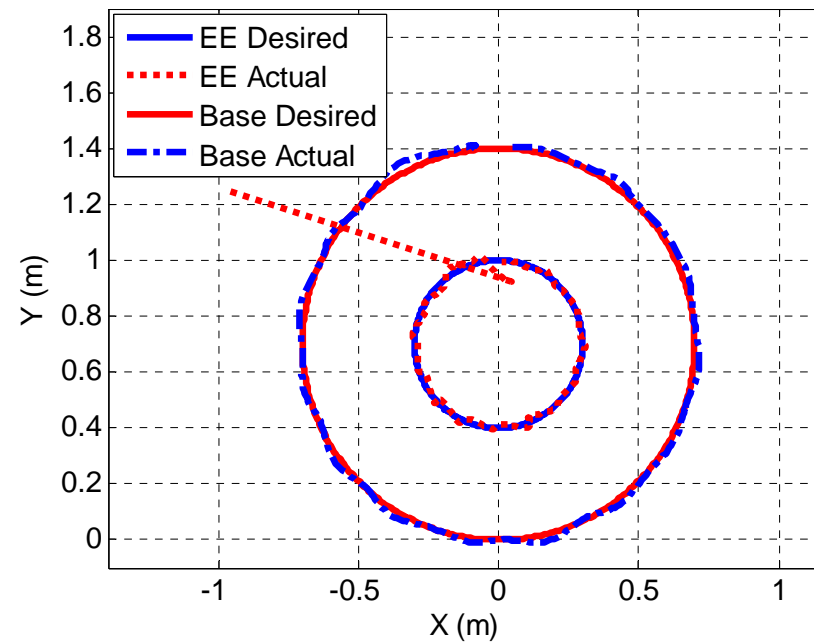
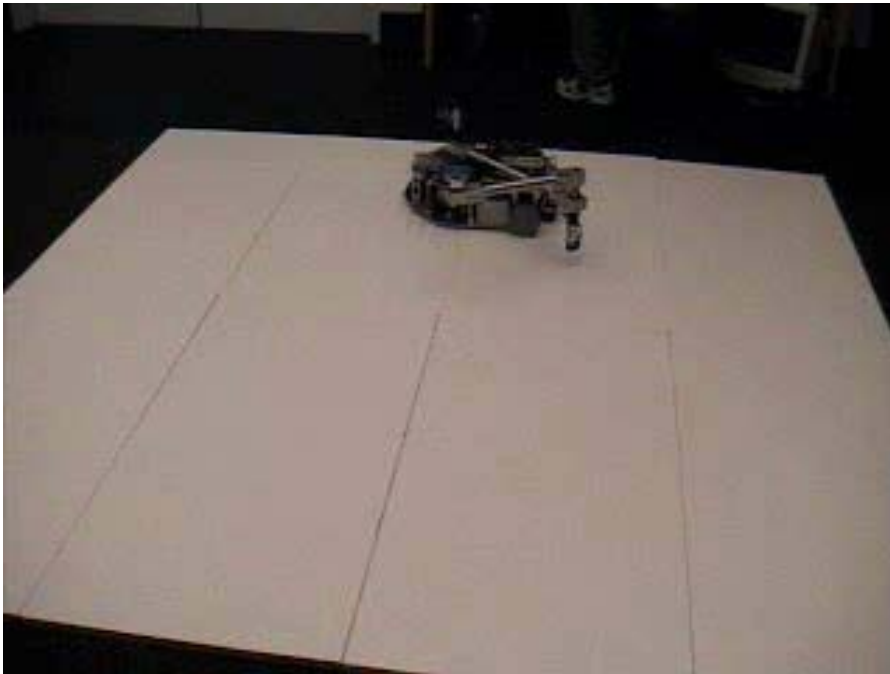
# Experimental Result I: Motion / Trajectory Tracking

*End-effector tracks sinusoidal trajectory*  
*Mobile base tracks a straight line*  
*No interaction force control*



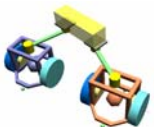
# Experimental Result II: Motion / Trajectory Tracking

**End-effector and mobile base  
tracks different circular trajectory**  
*No interaction force control*



# Discussion

- Verified the capability of the developed control framework to *resolve redundancy* and permit creation of a *decoupled end-effector impedance-mode controller*
- Demonstrated the ability to exploit the redundancy to *simultaneously follow complex end-effector* and *secondarily follow mobile base paths* (with a natural prioritization to end-effector tracking performance)
- Established the required *infrastructure* and *decouple dynamic control framework* to achieve decentralized payload manipulation by multiple mobile manipulator agents



**Thank You!**

